Simulation of Space Charge Waves in Finite Length High-Current Beams with Wall Resistivity

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To Professor Arnulf Schlüter on his 60th Birthday

Collective space charge effects play an important role in intense unneutralized beams of non relativistic heavy ions, which are of interest in recently proposed high-current accelerators (such as drivers for inertial confinement fusion etc.). Of particular importance is the propagation of wave-like perturbations including the destabilizing effect of dissipation due to finite electrical conductivity in the surrounding walls. The propagation characteristics of such waves are investigated by means of $2^1/_2$-dimensional computer simulation, with emphasis on dispersion, steepening (leading to solitary waves), and the problem of reflection of a growing wave at the beam tail with subsequent conversion into a damped wave propagating back into the beam. The net result on beam quality is discussed.

I. Introduction

High intensity beams of non-relativistic heavy ions have been suggested as driver to ignite thermonuclear targets using the scheme of inertial confinement fusion (for a recent review see Ref. [1]). The proposed multi-stage acceleration schemes employ r.f. linear accelerators with storage rings for current amplification or, alternatively, a high-current induction linear accelerator. The high current in these accelerators has necessitated the study of space charge effects in beam dynamics, in particular collective effects, which require plasma theoretical methods of description. The dynamical behaviour of such a "beam plasma" is strongly influenced by the screening effect of a close conducting beam pipe ("reduced" plasma frequency). Of major concern for an intense beam is the deterioration of beam quality due to a longitudinal bunching instability driven by wall resistivity. The instability was first analyzed by Nielsen et al. [2] and many subsequent workers for infinitely long (or circular) beams. The "slow" space charge wave (propagating backwards in the beam) is known to be a negative energy wave and thus becomes unstable in the presence of a sink of energy, like wall dissipation. On the contrary, the "fast" wave (propagating forwards) is a positive energy wave and damped by dissipation. The instability threshold in an endless beam is described by the well-known "Keil-Schnell" self-bunching criterion [3].

For an observer in the beam frame the instability is convective and the question can be raised whether reflection of a growing slow wave at the tail of a *finite length* bunch and its subsequent conversion into a damped fast wave would limit or even suppress the destabilizing effect of finite wall resistivity.

Indeed, analytic work on stability of a rectangular pulse shape bunch has indicated the existence of only stable eigenmodes [4 – 6], contrary to the infinitely long beam case. 1-dimensional computer simulation of the instability has shown the possibility of longitudinal beam break up at the tail, for large enough resistivity [7, 8]. Bisognano et al. [8] have pointed out the importance of dispersion (for wave lengths not large compared with the pipe radius) and nonlinear steepening of longitudinal space charge waves, and suggest that such a system could support solitary waves. This has necessitated the development of multi-dimensional simulation of a beam in a pipe in order to properly take into account the transverse dimension giving rise to dispersion.

The present study presents first results of such a multi-dimensional numerical treatment of resistively driven longitudinal waves in finite length particle beams. We observe that the phenomena presented are not restricted to intense ion beams; in fact they should be of interest also to intense electron beams employed in microwave tubes.

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II. Beam Model and Basic Parameters

In this section we outline those features and parameters of high-current beams, which define the physics of space charge waves. As usual in conventional particle accelerators beams are assumed to propagate in vacuum inside a conducting cylindrical pipe with no neutralizing charges present.

1. Long Beam in a Perfectly Conducting Pipe

Trajectories of ions with charge q, mass M and non relativistic velocity v_0 are given by the set of equations (in a frame of reference moving with v_0)

$$\frac{d^2 x}{dt^2} + \varkappa_x x - \frac{q}{M} E_x^s = 0\,, \tag{1}$$

$$\frac{d^2 y}{dt^2} + \varkappa_y y - \frac{q}{M} E_y^s = 0\,, \tag{2}$$

$$\frac{d^2 z}{dt^2} - \frac{q}{M}(E_0 + E_z^s) = 0\,. \tag{3}$$

The $\varkappa_{x,y}$ describe transverse magnetic focusing forces to compensate beam divergence and the repulsive electrostatic self-field components $E^s_{x,y}$, whereas the longitudinal component E_z^s is compensated by an externally supplied field $E_0 \cdot \boldsymbol{E}^s$ follows from Poisson's equation

$$\boldsymbol{\nabla} \cdot \boldsymbol{E}^s = \varrho/\varepsilon_0 \tag{4}$$

with boundary condition of zero potential on the beam pipe.

To characterize a stationary beam one has to find a self-consistent distribution in 6-dimensional phase space, which is usually possible only in an approximate sense by assuming decoupling between longitudinal and transverse phase planes. For beams that are long compared with the pipe diameter it is of great help to use the long-wavelength formula for E_z^s

$$E_z^s = -\frac{g \cdot q}{4 \pi \varepsilon_0} \cdot \frac{\partial n}{\partial z} \tag{5}$$

with n the particles per unit length (line density) and g a geometry factor, which is

$$g = 1 + 2 \ln (R_w/R_b) \tag{6}$$

for a beam with radius R_b in a pipe with radius R_w. This geometry factor describes the screening of the electric self-field by a nearby conducting wall; i. e. electric field lines are strongly bent to the wall.

In order to hold a stationary bunched beam an external field E_0 is required which has to compensate E_z^s and in addition provide for a net restoring force to contain longitudinal thermal motion.

The assumption of high intensity implies that the electrostatic field energy in the bunch is considerably larger than the thermal energy (i. e. the kinetic energy of *relative* motion). In this limit collective motion takes place on a much shorter time-scale than single particle or thermal motion.

2. Space Charge Waves of a Cold Beam

As an example of collective motion we give a brief account of elementary properties of longitudinal waves driven by space charge. In the limit of an intense beam we neglect thermal motion to first approximation. This cold beam plasma can be described by hydrodynamic equations, rather than using the full Vlasov equation as in the case of a warm or finite emittance beam. Using (5) we obtain

$$\frac{\partial n}{\partial t} + \frac{\partial (n\, v)}{\partial z} = 0\,, \tag{7}$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} + \frac{q^2 g}{4 \pi \varepsilon_0 M} \frac{\partial n}{\partial z} = 0 \tag{8}$$

with v the macroscopic streaming velocity in the beam frame.

By linearizing (7, 8) for a sinusoidal perturbation we obtain the phase velocity of a wave in a beam given by ($k = 2\pi/\lambda$)

$$v_b{}^2 = \omega^2/k^2 = \frac{g\, R_b{}^2}{4} \omega_p{}^2 \tag{9}$$

with $\omega_p = (q^2 n/\varepsilon_0 M)^{1/2}$ the plasma frequency and $\omega^2 = k^2 (g\, R_b{}^2/4)\, \omega_p{}^2$ the "reduced" plasma frequency due to wall screening. We observe that the absence of dispersion expressed by (9) is true only for long wave lengths, i. e. $k R_b \ll 1$. For $k R_b \sim 1$, (5) becomes invalid – a well-known fact in microwave tube design [9] – and there is strong dispersion shown in Fig. 1; in the limit $k \cdot R_b \gg 1$ we obtain the usual wave length independent oscillation frequency $\omega = \omega_p$.

Bisognano et al. [8] have pointed out that dispersion can be described approximately by adding to (8) the next expansion term in (5), which is proportional to $\partial^3 n/\partial z^3$. The resulting equation is related to the Korteweg-de Vries equation and

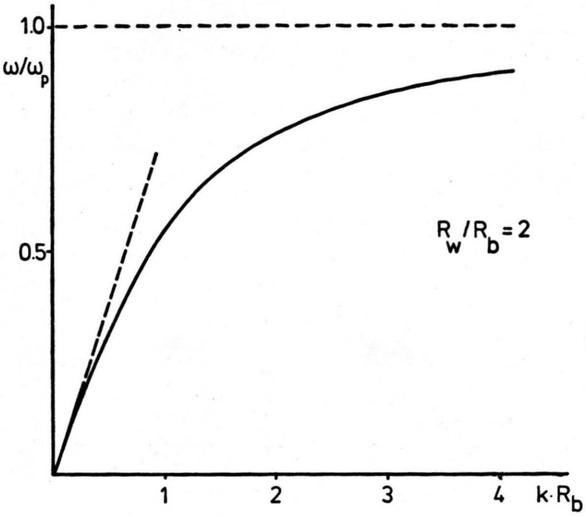

Fig. 1. Plasma frequency reduction factor ω/ω_p versus normalized wave number $k R_b$ for a beam in a conducting tube with circular cross section and $R_w/R_b=2$. In the long wavelength limit $k R_b \ll 1$ the linear dispersion law $\omega/\omega_p = (\sqrt{g}/2)\, k R_b$ is approached, whereas for $k R_b \gg 1$ we have $\omega \to \omega_p$.

similar non-linear features may be expected. Ignoring the self-force, the reduced non-linear wave equation

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = 0 \qquad (10)$$

allows for the implicit wave solution

$$v = v(z - v\, t) \qquad (11)$$

which indicates large wave velocity ($\approx v_{max}$) at points where the wave amplitude is large. If this steepening at the wave front is balanced by dispersion – for wave lengths of the order of the pipe diameter – solitary waves may be supported by the system.

3. *Instability due to Finite Resistivity*

The simplest model of taking into account wall resistivity and the impedance of accelerating gaps is to assume a resistive drag force proportional to the current

$$E_z{}^r = R \cdot I(z, t) \qquad (12)$$

with $R[\Omega/\mathrm{m}]$ the resistivity per unit length and $I = v_0 \cdot n$. With this extra force we obtain in the cold beam limit and for $R \lesssim k\, g$

$$\Re e\, \omega = \pm v_b \cdot k\,, \qquad (13)$$

$$\Im m\, \omega = \tfrac{1}{2} v_b \cdot R/g\,. \qquad (14)$$

The forward (fast) wave is damped and the backward (slow) wave is growing in an infinitely long beam. The linearized analysis breaks down if a bunch of finite length is considered and a space charge wave approaches the bunch end, where the unperturbed density drops to zero.

An analytic treatment of resistively growing waves that propagate into a region of vanishing zero order density has not been attempted so far. Several authors have considered eigenmodes for special bunch profiles amenable to analytic treatment and find no unstable eigenfrequencies, but spatial amplification from pulse head to tail [4 – 6]. This work has indicated a potential danger to beam quality and has necessitated multi-dimensional computer simulation to investigate the questions of conversion of a growing mode into a damped mode, non-linear propagation characteristics and the resulting consequences on beam quality.

III. Computer Simulation

1. *Method and Beam Model*

The simulation is performed with a particle-in-cell code. Particles are traced in the 6-dimensional phase using Cartesian geometry. Poisson's equation is solved with a fast Poisson solver on a rectangular area-weighted mesh with 16×352 cells in r, z consistent with a cylindrically symmetric beam. On the radial tube wall perfect electrical conductivity is assumed. In longitudinal direction periodic boundary conditions have been applied. Finite resistivity is taken into account by an extra force according to (12). The initial load is statistic with a filling of the 6-dimensional phase space such that the radial density is uniform and fills half the tube aperture; the longitudinal velocity distribution (in the beam frame) is Gaussian and the line density constant for the infinitely long beam and parabolic for the bunched beam. An initial highly localized bump of the line density is generated in the beam center, which gives rise to a forward and a backward moving wave packet that can be easily distinguished from the noise. The initial beam distribution is relatively cold, i. e. we assumed a ratio of thermal velocity to wave velocity

$$\delta v_{th}/v_b = 0.2\,. \qquad (15)$$

2. *Results*

In Figs. 2 – 5 projections are shown of the 6-dimensional phase space distribution into the longitudinal-transverse configuration space $(z-x)$ and the longitudinal phase plane ($z-v_z$, with v_z in the beam frame of reference which is assumed to move to the right). The frames show $8 \cdot 10^3$ out of $32 \cdot 10^3$ simulation particles. The geometrical dimensions are given by $R_w/R_b=2$ and $L/R_b=80$, where L is the beam length. We observe that scales of length z and time T have been chosen arbitrarily. The physics is given by the following: $v_b \approx 3.7$ in all cases; $\delta v_{th}/v_b \approx 0.2$ and the number of resistive e-foldings for a small perturbation travelling from the bunch head to the tail.

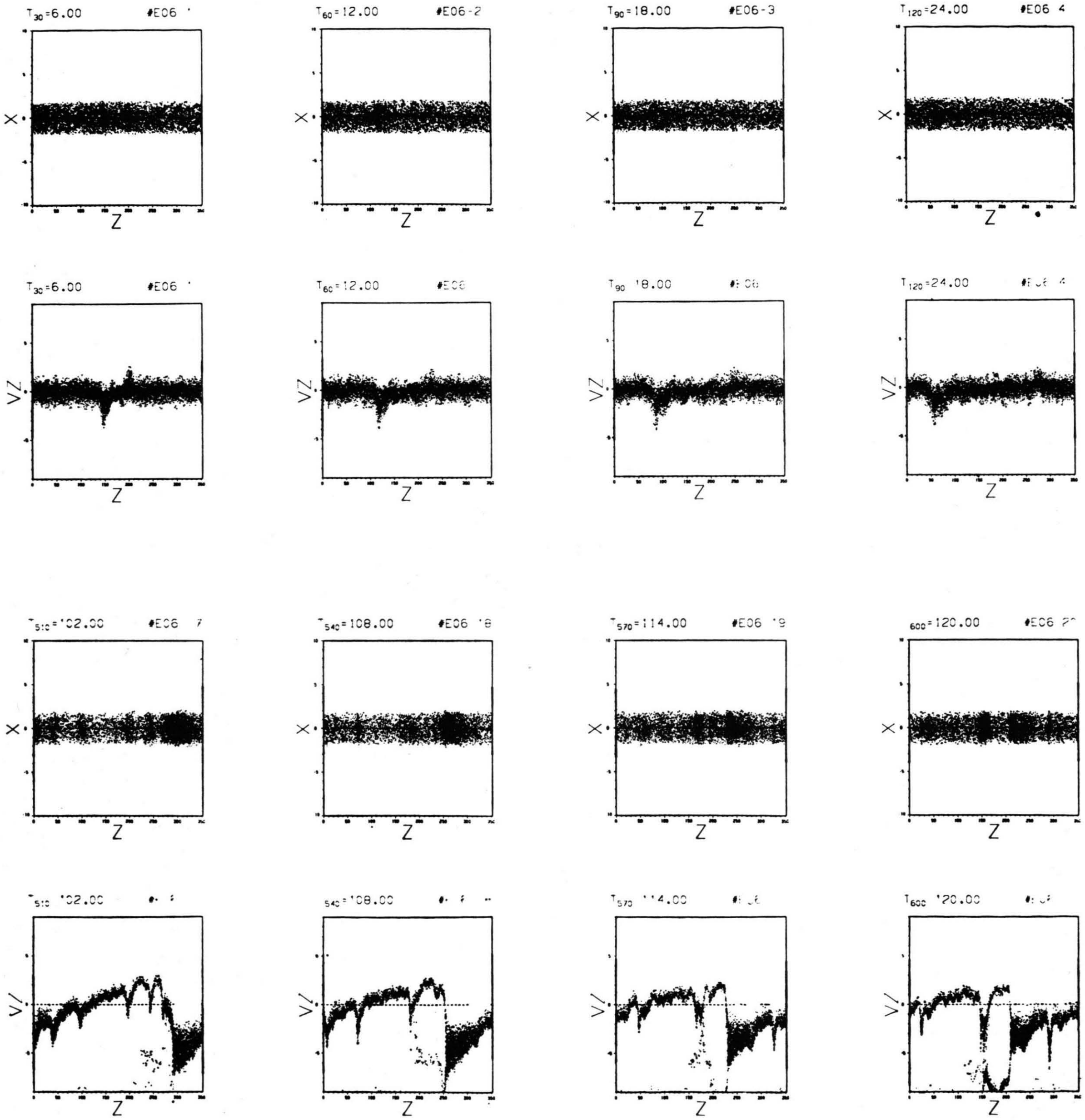

Fig. 2. Infinitely long beam with finite resistivity (1 e-folding during $\Delta T=20$). A backwards and a forwards traveling wave are excited in the beam center ($z=175$) at $T=0$. Projections into configuration space $z-x$ and longitudinal phase space $z-v_z$ at an early stage ($T=6 \dots 24$) and at a highly nonlinear stage ($T=102 \dots 120$) showing micro-bunching and steepened wave fronts.

a) Infinitely Long Beam

A constant line density with periodic boundaries at $z=0$, 350 is shown in Figure 2. The left going backward (slow) space charge wave is amplified by resistivity (after leaving the mesh at $z=0$ ($T \approx 36$) it re-enters at $z=350$). The forward wave is resistively damped and has almost disappeared at $T=20$, corresponding to one e-folding time. The second row of frames shows the highly nonlinear stage at $T \gtrsim 102$. Apart from the strong amplification of the initial noise fluctuations, the initial perturbation has grown to large amplitude and steepening has occurred. The wave front propagates at the velocity of the maximum amplitude of v_z supporting the conjecture of (11). This velocity is more than twice the small signal velocity given by v_b (neglecting dispersion). The simulation indicates rapid steepening as soon as the coherent velocity perturbation exceeds v_b, in which case particles are trapped in the wave potential and pile up in the wave front. The vortex developing at $T \approx 120$ is a result of electrostatic acceleration of particles (to negative v_z) at the sharp wave front and subsequent deceleration at the neighbouring wave maximum.

b) Bunched Beam

A zero resistivity case is shown in Figure 3. The backwards and forwards traveling waves are reflected at the bunch ends. The actual reflection is substantially delayed due to the phase change by π (note that beyond bunch ends the applied voltage is rising steeply, since it is not compensated by space charge). In spite of the noticeable dispersion indicated by Fig. 1 the wave packets do not decay even after several reflections, which indicates the importance of nonlinear effects.

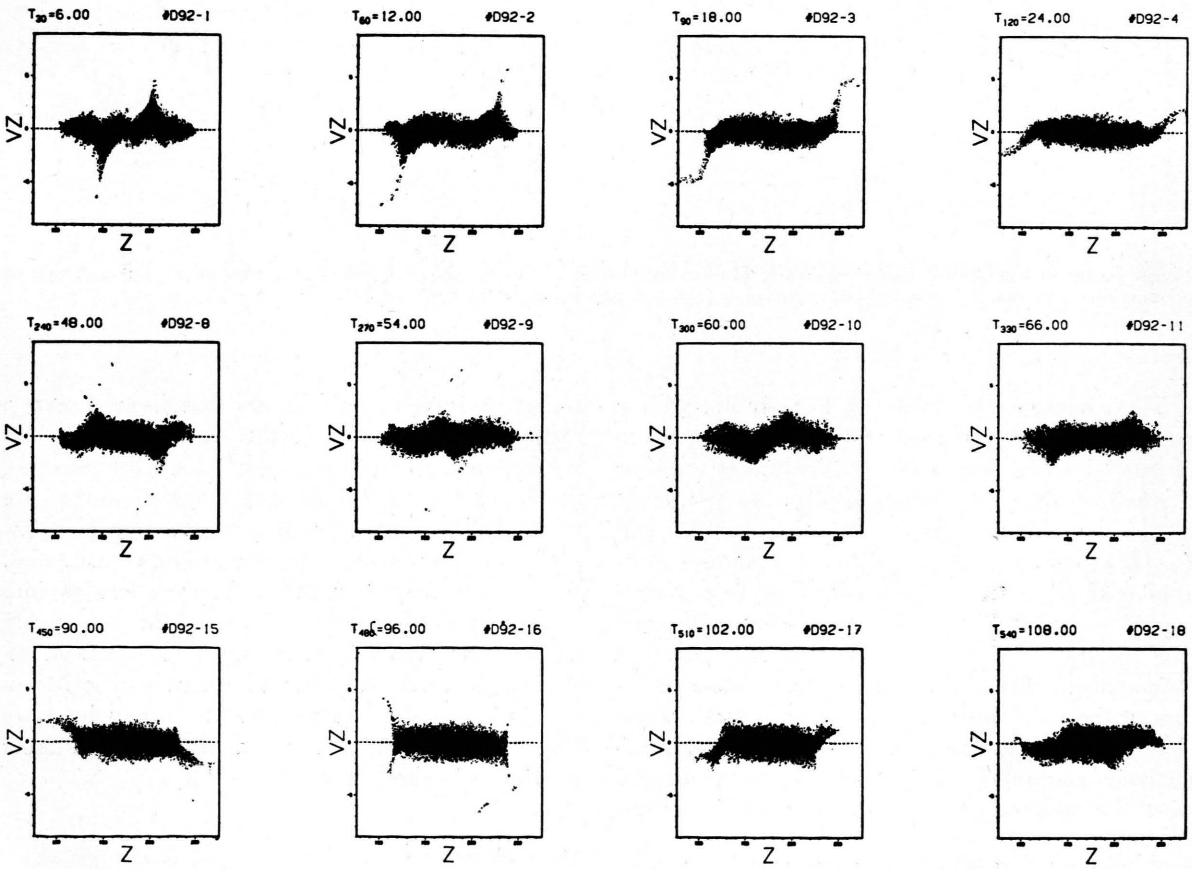

Fig. 3. Bunched beam without resistivity. Projections into longitudinal phase space $z - v_z$ showing three sequences of time intervals. The backwards and forwards traveling waves are reflected at the bunch ends (suffering a phase change of π) and travel in clockwise direction around the longitudinal beam distribution.

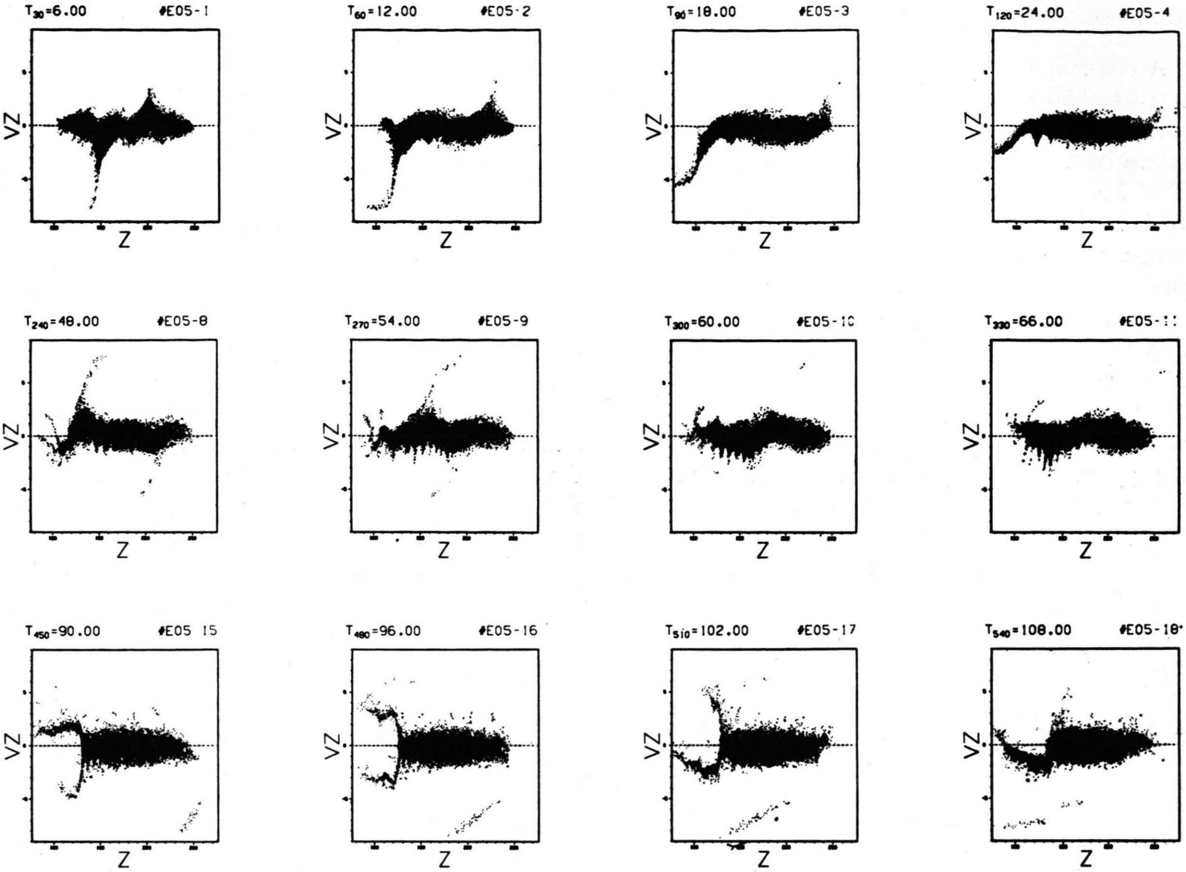

Fig. 4. Same as Fig. 3 with finite resistivity (1 e-folding during $\Delta T = 20$) showing the suppression of net growth due to conversion of growing (left-going) into damped (right-going) waves at bunch ends.

The same case is shown in Fig. 4, but with a resistivity that corresponds to a linearized growth of one e-folding during $\Delta T = 20$, which is the time a small signal perturbation requires to propagate from the bunch center to the bunch end ($\sim [L/2]/v_b$). Even at $T = 120$ the picture is very similar to the previous zero resistivity case. Short-wave length fluctuations are slightly amplified, and at $T \gtrsim 100$ there is a small fraction of particles at increased positive v_z, which is due to wave reflection at the tail. This case demonstrates that conversion into damped modes at the bunch tail can effectively suppress resistive growth, as compared with the unbunched beam of Fig. 2 (same resistivity).

The situation changes drastically if the resistivity is doubled as shown in Fig. 5, hence two e-foldings occur during $\Delta T = 20$. The initial coherent perturbation in the bunch center has been chosen a factor of e smaller than in the previous case so that it assumes about the same amplitude when reaching the beam tail; the dominant effect, however, are relatively short wave length perturbations (λ a few pipe diameters) which saturate at large amplitudes. Evidently the conversion of growing modes into damped modes is rather ineffective at this highly nonlinear level due to the loss of coherence at reflection, hence broadening of the velocity distribution is inevitable. The average δv_{th} grows by more than a factor of 2 until increased Landau damping terminates further growth.

IV. Conclusions

By means of computer simulation we have shown that the propagation of large amplitude space charge

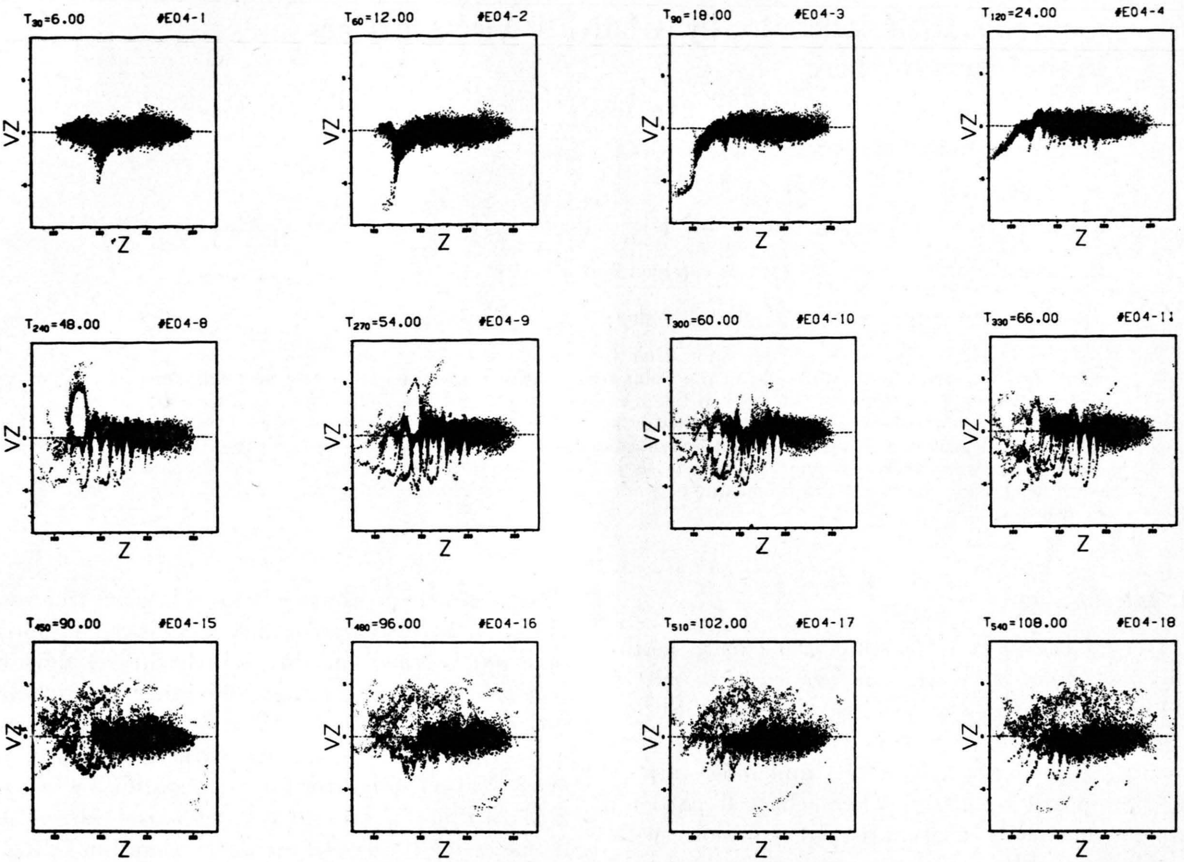

Fig. 5. Same as Fig. 4 with doubled resistivity (2 e-foldings during $\Delta T = 20$) but a factor e smaller initial coherent wave amplitudes. Wave conversion at bunch ends is insufficient due to increased nonlinearity, and beam quality is substantially deteriorated.

waves in long beams in a conducting pipe is characterized by a wave speed increasing with the amplitude and by steepening. Wall dissipation leading to wave amplification in long unbunched beams has been found to destabilize bunched beams only if resistivity is sufficiently large. The critical value is the number of e-foldings that occur during a single transit of a small perturbation across the beam length. The instability results in micro-bunching and an increased momentum width of the beam.

Acknowledgement

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